| ROLL NUMBER | | | | |
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| SET | 1 |
|---|---|

QP.Code:041/01/1

# INDIAN SCHOOL MUSCAT
## FIRST PRE BOARD EXAMINATION
## MATHEMATICS (041)

CLASS : XII
DATE: 15.01.2023

TIME ALLOTED : 3 HRS.
MAXIMUM MARKS: 80

***GENERAL INSTRUCTIONS:***

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's** and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)**-type questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)**-type questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)**-type questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

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**SECTION – A (Questions 1 to 20 carry 1 mark each)**

1. The integrating factor of the differential equation $\frac{dy}{dx} + y\,tanx - secx = 0$ is

   (A) $\tan x$ (B) $sec^2x$ (C) $\sec x$ (D) $\frac{tan^2x}{2}$

2. If $A = [a_{ij}]_{m\times n}$ is a square matrix, then which of the following is true?

   (A) $m < n$ (B) $m > n$ (C) $m = n$ (D) $m = 0$

3. $\int \frac{sin^2x - cos^2x}{sin^2x\, cos^2x}\, dx$ is equal to

   (A) $tanx + cosec\, x + C$ (B) $tanx + cot\, x + C$

   (C) $tanx - cot\, x + C$ (D) $tanx + sec\, x + C$

4. The value of $(\hat{k} \times \hat{j}).\hat{\imath} + (\hat{j}.\hat{k}) + 3$

   (A) 0 (B) 1 (C) 2 (D) 3

5. If $A$ and $B$ are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A/B) = \frac{1}{4}$, then $P(A' \cap B')$ equals

   (A) $\frac{1}{12}$ (B) $\frac{3}{4}$ (C) $\frac{1}{4}$ (D) $\frac{3}{16}$

6. A point that lies on the line $\frac{x-1}{-2}=\frac{y+3}{4}=\frac{1-z}{7}$ is:

(A) (1, -3, 1) (B) (-2, 4, 7) (C) (-1, 3, 1) (D)(2, -4, -7)

7. If $A=[a_{ij}]$ be a skew-symmetric matrix of order $n$, then

(A) $a_{ij}=\frac{1}{a_{ji}}$ for all $i, j$ (B) $a_{ij}\neq 0$ for all $i, j$

(C) $a_{ij}=0$, where $i=j$ (D) $a_{ij}\neq 0$, where $i=j$

8. A man is watching an aero plane which is at the coordinate point A (4, -1, 3), assuming the man is at O (0, 0, 0). At the same time, he saw a bird at coordinate point B (2, 0, 4). The unit vector along $\overrightarrow{AB}$ is

(A) $\frac{2}{6}\hat{\imath}+\frac{1}{6}\hat{\jmath}+\frac{1}{6}\hat{k}$ (B) $\frac{-2}{\sqrt{6}}\hat{\imath}+\frac{1}{6}\hat{\jmath}+\frac{1}{6}\hat{k}$

(C) $\frac{-2}{\sqrt{6}}\hat{\imath}+\frac{1}{\sqrt{6}}\hat{\jmath}+\frac{1}{\sqrt{6}}\hat{k}$ (D) $\frac{4}{\sqrt{6}}\hat{\imath}+\frac{2}{\sqrt{6}}\hat{\jmath}+\frac{3}{\sqrt{6}}\hat{k}$

9. Let $A$ be a $3\times 3$ matrix such that $C_{11}, C_{12}, C_{13}$ are the cofactors of $a_{11}, a_{12}, a_{13}$ respectively. What is the value of $a_{11}C_{11}+a_{12}C_{12}+a_{13}C_{13}$ ?

(A) 0 (B)1 (C) $-|A|$ (D) $|A|$

10. $\int e^x\sqrt{e^x+5}\ dx=$

(A) $\frac{3}{2}(e^x+5)^{3/2}+C$ (B) $\frac{3}{2}(e^x+5)^{2/3}+C$

(C) $\frac{2}{3}(e^x+5)^{3/2}+C$ (D) $\frac{2}{3}\sqrt{e^x+5}+C$

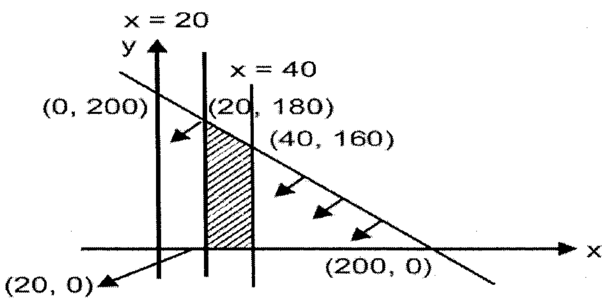
11. If $A$ is a matrix of order $m\times n$ and $B$ is a matrix such that $AB'$ and $B'A$ are both defined, then the order of matrix $B'$ is

(A) $m\times n$ (B) $m\times m$ (C) $n\times n$ (D) $n\times m$

12. The order and the degree of the differential equation $2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$ are:

(A) 1, 1 (B) 2, 1 (C) 1, 2 (D) 3, 1

13. For an L.P.P. the objective function is Z = 400x + 300y, and the feasible region determined by a set of constraints (linear inequalities) is shown in the graph.



Find the coordinates at which the objective function is maximum.

(A)(20, 0) (B) (40, 0) (C) (40, 160) (D) (20, 180)

14. If $f(x) = x\ \tan^{-1} x,$ then $f'(1) =$

(A) $1 + \frac{\pi}{4}$ (B) $\frac{1}{2} + \frac{\pi}{4}$ (C) $\frac{1}{2} - \frac{\pi}{4}$ (D) 2

15. The greatest integer function defined by $f(x) = [x], 0 < x < 2$ is not differentiable at

(A) $x = 0$ (B) $x = 1$ (C) $x = 2$ (D) any point

16. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then the value of $x$ is

(A) $\pm 6$ (B) 3 (C) 6 (D) $\pm 3$

17. If $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{b}$ is a vector such that $\vec{a}.\vec{b} = |\vec{b}|^2$ and $|\vec{a} - \vec{b}| = \sqrt{7}$, then $|\vec{b}| =$

(A)$\sqrt{7}$ (B) $\sqrt{3}$ (C)7 (D)3

18. The point which does not lie in the half-plane $2x + 3y - 12 \leq 0$ is

(A)( 1, 2) (B) ( 2, 3) (C)( 2, 1) (D) ( -3, 2)

**ASSERTION-REASON BASED QUESTIONS**

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both **A** and **R** are true and **R** is the correct explanation of **A**.
(b) Both **A** and **R** are true but **R** is not the correct explanation of **A**.
(c) **A** is true but **R** is false.
(d) **A** is false but **R** is true.

19. **Assertion (A) :** $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \frac{2\pi}{3}$

**Reason (R) :** $\sin^{-1}(\sin\theta) = \theta$, if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

20. **Assertion (A) :** The angle between the lines whose direction cosines are

$$-\frac{\sqrt{3}}{4}, \frac{1}{4}, -\frac{\sqrt{3}}{2} \text{ and } -\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2} \text{ is } 120°.$$

**Reason (R) :** The angle between the lines whose direction cosines are $l_1, m_1, n_1$ and $l_2, m_2, n_2$ is given by $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$.

**SECTION – B (Questions 21 to 25 carry 2 marks each)**

21. Find the intervals in which the function $f(x) = 2x^3 - 24x + 107$ is

(i) strictly increasing (ii) strictly decreasing

22. Find the projection of vector $(\vec{b} + \vec{c})$ on $\vec{a}$, where $\vec{a} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$ .

23. Find the value of : $sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$.

24. If $f(x) = \sqrt{1 + cos^2(x^2)}$ , find $f'\left(\frac{\sqrt{\pi}}{2}\right)$.

OR

If $f(x) = \tan^{-1}\left(\frac{\sqrt{x^2+1}-1}{x}\right)$ , find $f'(x)$.

25. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular. Find the angle $\theta$ between $\vec{a}$ and $\vec{b}$ .

(OR)

Using direction ratios, show that the points A(2,3,4), B(-1,-2,1) and C(5,8,7) are collinear.

**SECTION – C (Questions 26 to 31 carry 3 marks each)**

26. Find the particular solution of the differential equation: $\frac{dy}{dx} + 2y\ tanx = sin\ x$, given that $y = 0$ when $x = \frac{\pi}{3}$

(OR)

Solve the following differential equation: $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + xdy = 0$

27. Evaluate $\int_0^{\pi} \frac{x\ sinx}{1+cos^2x}\,dx$ using properties of definite integrals.

28. Amit and Nisha appear for an interview for two vacancies in a company. The probability of Amit's selection is $\frac{1}{5}$ and that of Nisha's selection is $\frac{1}{6}$ .What is the probability that

(i) only one of them is selected? (ii) none of them are selected ?

(OR)

Find the mean number of defective items if two items are drawn one-by-one without replacement from an urn containing 6 items, which include 2 defective items?

29. Evaluate: $I = \int_1^4 [|x-1| + |x-2| + |x-4|]\ dx$

30. Solve the following Linear Programming Problem graphically:

Maximize Z = $x + y$, subject to the constraints

$\frac{x}{25} + \frac{y}{40} \le 1;\ 2x + 5y \le 100, x \ge 0, y \ge 0.$

31. Evaluate: $\int \frac{3x+1}{(x-1)^2(x+3)}dx$ (OR) Evaluate : $\int \left\{log(\log x) + \frac{1}{(log x)^2}\right\} dx$

**SECTION – D (Questions 32 to 35 carry 5 marks each)**

32. Prove that the relation $R$ in the set $A = \{5, 6, 7, 8, 9\}$ given by

$R = \{(a, b): |a - b|\ is\ divisible\ by\ 2\}$ is an equivalence relation. Find all elements related to element 6.

(OR)

Let $f: \mathbf{N} \rightarrow \mathbf{N}$ be defined by

$f(x) = \begin{cases} x+1, if\ x\ is\ odd \\ x-1, if\ x\ is\ even \end{cases}$ for all $x \in \mathbf{N}$. Show that $f$ is bijective.

33. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the straight line $3x + 4y = 12$.

34. Find the adjoint of the matrix $A = \begin{bmatrix} -1 & -1 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that $A.(adj\ A) = |A|I_3$, where $I_3$ is the identity matrix of order 3.

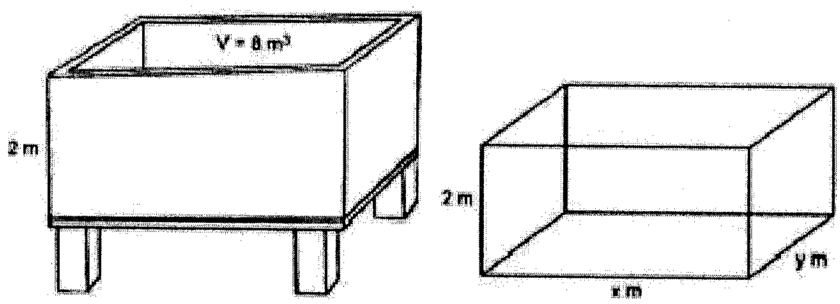
35. By computing the shortest distance, prove that the lines $\vec{r} = (\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda(3\hat{\imath} - \hat{\jmath})$ and $\vec{r} = (4\hat{\imath} - \hat{k}) + \mu(2\hat{\imath} + 3\hat{k})$ intersect. Also find their point of intersection.

(OR)

The scalar product of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of vectors $\vec{b} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\vec{c} = \lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to 1. Find the value of $\lambda$.

**SECTION – E- CASE STUDY (Questions 36 to 38 carry 4 marks each)**

36.




On the request of villagers, a construction agency designs a tank with the help of an architect. Tank consists of a rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is 8 $m^3$. The construction of the tank costs ₹ 70 per sq.m for the base and ₹ 45 per sq.m for sides.

(a) If $x$ and $y$ represent the length and breadth of the rectangular base, find the relation between the variables.

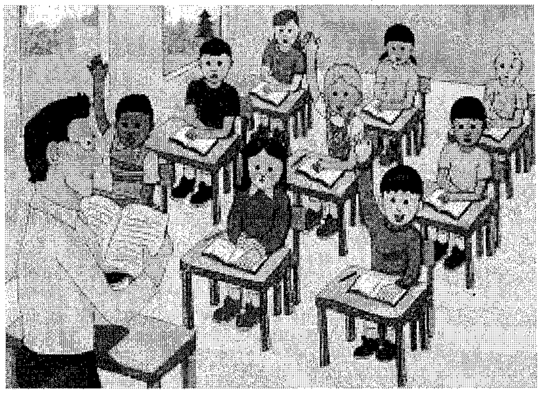
(b) Express the cost C of making the tank in terms of $x$.

(c) Find the value of $x$ so that the cost of construction is minimum.

(OR)

(c) Verify by second derivative test that cost is minimum at a critical point.

37.



There are three categories of students in a class of 60 students :

A : Very hardworking students

B : Regular but not so hardworking

C : Careless and irregular

It is known that 10 students are in Category A, 30 in Category B and the rest in Category C. It is also found that the probability of students of Category A, unable to get good marks in the final exam, is 0.002, of Category B it is 0.02 and of Category C, this probability is 0.20.

(a) If a student selected at random was found to be the one who could not get good marks in the exam, find the probability that this student is of Category C.

(b) Find the probability that the selected student is unable to get good marks in the exam.

(c) A student selected at random was found to be the one who could not get good marks in the exam, then find the probability that this student is NOT of Category A.

(OR)

(c) If a student selected at random was found to be the one who could not get good marks in the exam, find the probability that this student is of Category B.

38.


Assume that a spherical raindrop evaporates at a rate proportional to its surface area, given by the differential equation $\frac{dV}{dt} = kS$, where $V$ is the volume and $S$ is the surface area of the spherical raindrop and $k$ is a constant.

(a) If its radius originally is 3 mm, establish a relation between the radius and time $t$.

(b) After 1 hour, if the radius has been reduced to 2 mm, find the radius of the raindrop at any time $t$.

******END OF THE QUESTION PAPER******

| ROLL NUMBER | | | | |
|---|---|---|---|---|

| SET | 2 |
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QP.Code:041/01/2

# INDIAN SCHOOL MUSCAT
# FIRST PRE BOARD EXAMINATION 2023
# MATHEMATICS (041)

CLASS : XII
DATE: 15.01.2023

TIME ALLOTED : 3 HRS.
MAXIMUM MARKS: 80

***GENERAL INSTRUCTIONS:***

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's** and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)**-type questions of 2 mark each.
4. **Section C** has 6 **Short Answer (SA)**-type questions of 3 mark each.
5. **Section D** has 4 **Long Answer (LA)**-type questions of 5 mark each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

## SECTION – A (Questions 1 to 20 carry 1 mark each)

1. If $A = [a_{ij}]$ be a skew-symmetric matrix of order $n$, then

(A) $a_{ij} = \frac{1}{a_{ji}}$ for all $i, j$ (B) $a_{ij} \neq 0$ for all $i, j$

(C) $a_{ij} = 0$, where $i = j$ (D) $a_{ij} \neq 0$, where $i = j$

2. If $A$ and $B$ are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A/B) = \frac{1}{4}$, then $P(A' \cap B')$ equals

(A) $\frac{1}{12}$ (B) $\frac{3}{4}$ (C) $\frac{1}{4}$ (D) $\frac{3}{16}$

3. $\int \frac{sin^2x - cos^2x}{sin^2x\, cos^2x}\, dx$ is equal to

(A) $tanx + cosec\, x + C$ (B) $tanx + cot\, x + C$

(C) $tanx - cot\, x + C$ (D) $tanx + sec\, x + C$

4. A man is watching an aero plane which is at the coordinate point A (4, -1, 3), assuming the man is at O (0, 0, 0). At the same time, he saw a bird at coordinate point B (2, 0, 4). The unit

vector along $\overrightarrow{AB}$ is

(A) $\frac{2}{6}\hat{\imath}+\frac{1}{6}\hat{\jmath}+\frac{1}{6}\hat{k}$ (B) $\frac{-2}{\sqrt{6}}\hat{\imath}+\frac{1}{6}\hat{\jmath}+\frac{1}{6}\hat{k}$

(C) $\frac{-2}{\sqrt{6}}\hat{\imath}+\frac{1}{\sqrt{6}}\hat{\jmath}+\frac{1}{\sqrt{6}}\hat{k}$ (D) $\frac{4}{\sqrt{6}}\hat{\imath}+\frac{2}{\sqrt{6}}\hat{\jmath}+\frac{3}{\sqrt{6}}\hat{k}$

5. The order and the degree of the differential equation $2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$ are:

(A) 1, 1 (B) 2, 1 (C) 1, 2 (D) 3, 1

6. The value of $(\hat{k}\times\hat{\jmath}).\hat{\imath}+(\hat{\jmath}.\hat{k})+3$

(A) 0 (B) 1 (C) 2 (D) 3

7. Let $A$ be a $3\times 3$ matrix such that $C_{11}, C_{12}, C_{13}$ are the cofactors of $a_{11}, a_{12}, a_{13}$ respectively. What is the value of $a_{11}C_{11}+a_{12}C_{12}+a_{13}C_{13}$ ?

(A) 0 (B)1 (C) $-|A|$ (D) $|A|$

8. The greatest integer function defined by $f(x)=[x], 0<x<2$is not differentiable at

(A) $x=0$ (B) $x=1$ (C) $x=2$ (D) any point

9. If $A$ is a matrix of order $m\times n$ and $B$ is a matrix such that $AB'$ and $B'A$ are both defined, then the order of matrix $B'$ is

(A) $m\times n$ (B) $m\times m$ (C) $n\times n$ (D) $n\times m$

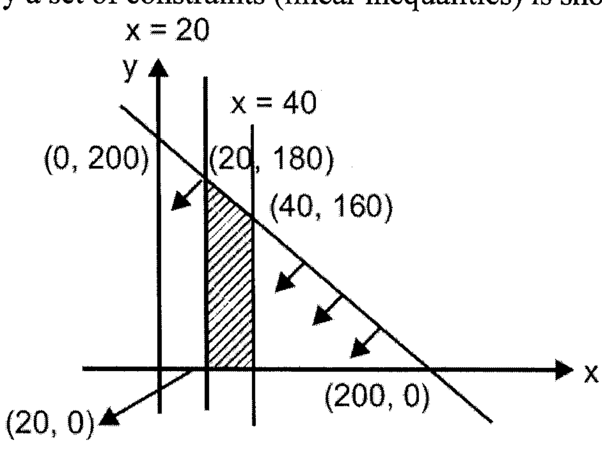
10. The integrating factor of the differential equation $\frac{dy}{dx}+y\,tanx-secx=0$ is

(A) $\tan x$(B) $sec^2x$ (C) $\sec x$ (D)$\frac{tan^2x}{2}$

11. A point that lies on the line $\frac{x-1}{-2}=\frac{y+3}{4}=\frac{1-z}{7}$ is:

(A) (1, -3, 1) (B) (-2, 4, 7) (C) (-1, 3, 1) (D)(2, -4, -7)

12. For an L.P.P. the objective function is Z = 400x + 300y, and the feasible region determined by a set of constraints (linear inequalities) is shown in the graph.




Find the coordinates at which the objective function is maximum.

(A)(20, 0) (B) (40, 0) (C) (40, 160) (D) (20, 180)

13. If $A = [a_{ij}]_{m\times n}$ is a square matrix, then which of the following is true?

(A) $m < n$ (B) $m > n$ (C) $m = n$ (D) $m = 0$

14. $\int e^x\sqrt{e^x+5}\ dx =$

(A) $\frac{3}{2}(e^x+5)^{3/2}+C$ (B) $\frac{3}{2}(e^x+5)^{2/3}+C$

(C) $\frac{2}{3}(e^x+5)^{3/2}+C$ (D) $\frac{2}{3}\sqrt{e^x+5}+C$

15. The greatest integer function defined by $f(x) = [x], 0 < x < 2$is not differentiable at

(A) $x = 0$ (B) $x = 1$ (C) $x = 2$ (D) any point

16. If $f(x) = x\ \tan^{-1}x$, then$f'(1) =$

(A) $1+\frac{\pi}{4}$ (B) $\frac{1}{2}+\frac{\pi}{4}$ (C) $\frac{1}{2}-\frac{\pi}{4}$ (D) 2

17. The point which does not lie in the half-plane $2x + 3y - 12 \le 0$ is

(A)( 1, 2) (B) ( 2, 3) (C)( 2, 1) (D) ( -3, 2)

18. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then the value of $x$ is

(A) $\pm 6$ (B) 3 (C) 6 (D) $\pm 3$

**ASSERTION-REASON BASED QUESTIONS**

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both **A** and **R** are true and **R** is the correct explanation of **A**.
(b) Both **A** and **R** are true but **R** is not the correct explanation of **A**.
(c) **A** is true but **R** is false.
(d) **A** is false but **R** is true.

19. **Assertion (A) :** The angle between the lines whose direction cosines are

$$-\frac{\sqrt{3}}{4}, \frac{1}{4}, -\frac{\sqrt{3}}{2} \text{ and } -\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2} \text{ is } 120°.$$

**Reason (R) :** The angle between the lines whose direction cosines are $l_1, m_1, n_1$ and $l_2, m_2, n_2$is given by $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$.

20. **Assertion (A) :** $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \frac{2\pi}{3}$

**Reason (R) :** $\sin^{-1}(\sin\theta) = \theta$, if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

**SECTION – B (Questions 21 to 25 carry 2 marks each)**

21. If $f(x) = \sqrt{1+sin^2(x^2)}$, find $f'\left(\frac{\sqrt{\pi}}{2}\right)$.

22. Find the absolute maximum and minimum values of the function $f$ given by

$f(x) = \frac{1}{3}x^3 - 3x^2 + 5x + 8$ in $[0, 6]$.

23. Find the value of : $sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$.

(OR)

Express $\tan^{-1}\left(\frac{\cos x}{1-sinx}\right), -\frac{\pi}{2} < x < \frac{\pi}{2}$, in the simplest form.

24. If the points $(x, -2), (5, 2)$ and $(8, 8)$ are collinear, find $x$ using determinants.

(OR)

Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular. Find the angle $\theta$ between $\vec{a}$ and $\vec{b}$.

25. Find the projection of vector$(\vec{b} + \vec{c})$ on $\vec{a}$, where $\vec{a} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$.

**SECTION – C (Questions 26 to 31 carry 3 marks each)**

26. Amit and Nisha appear for an interview for two vacancies in a company. The probability of Amit's selection is $\frac{1}{5}$ and that of Nisha's selection is $\frac{1}{6}$. What is the probability that

(i) only one of them is selected? (ii) none of them are selected ?

(OR)

Find the mean number of defective items if two items are drawn one-by-one without replacement from an urn containing 6 items, which include 2 defective items?

27. Solve the differential equation $e^{y/x} - y\, sin\left(\frac{y}{x}\right) + x\,\frac{dy}{dx}\, sin\left(\frac{y}{x}\right) = 0$

(OR)

Find the particular solution of the differential equation $\frac{dy}{dx} + y\, cotx = 2x + x^2 \cot x$, given that $y = 0$ when $= \frac{\pi}{2}$.

28. Solve the following Linear Programming Problem graphically:
Maximize $Z = x + y$, subject to the constraints
$\frac{x}{25} + \frac{y}{40} \leq 1;\ 2x + 5y \leq 100,\ x \geq 0, y \geq 0.$

29. Evaluate: $\int \frac{3x+1}{(x-1)^2(x+3)}dx$ (OR) Evaluate : $\int \left\{log(\log x) + \frac{1}{(logx)^2}\right\} dx$

30. Evaluate $\int_0^{\pi} \frac{xtanx}{secx+tanx}\, dx$ using properties of definite integrals.

31. Evaluate: $\int_1^5 [|x-1|+|x-2|+|x-3|]\ dx$.

## SECTION – D (Questions 32 to 35 carry 5 marks each)

32. Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Also find their point of intersection.

(OR)

The scalar product of the vector $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$ with a unit vector along the sum of vectors $\vec{b}=2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ and $\vec{c}=\lambda\hat{\imath}+2\hat{\jmath}+3\hat{k}$ is equal to 1. Find the value of $\lambda$.

33. Find the adjoint of the matrix $A=\begin{bmatrix}-1 & -1 & -2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{bmatrix}$ and hence show that $A.(adj\ A)=|A|\ I_3$ , where $I_3$ is the identity matrix of order 3.

34. Prove that the relation $R$ in the set $A=\{5,6,7,8,9\}$ given by $R=\{(a,b): |a-b|\ is\ divisible\ by\ 2\}$ is an equivalence relation. Find all elements related to element 6.

(OR)

Let $f$: **N** →**N** be defined by
$f(x)=\begin{cases}x+1, if\ x\ is\ odd\\ x-1, if\ x\ is\ even\end{cases}$ for all$x$ ∈ N. Show that $f$ is bijective.

35. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the straight line $2x+3y=6$.

## SECTION – E- CASE STUDY (Questions 36 to 38 carry 4 marks each)

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Assume that a spherical raindrop evaporates at a rate proportional to its surface area, given by the differential equation $\frac{dV}{dt}=kS$, where $V$ is the volume and $S$ is the surface area of the spherical raindrop and $k$ is a constant.

(a) If its radius originally is 3 mm, establish a relation between the radius and time $t$.

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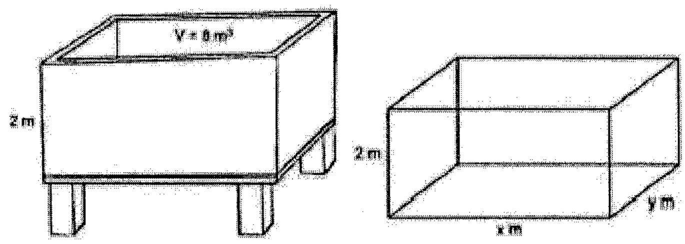


On the request of villagers, a construction agency designs a tank with the help of an architect. Tank consists of a rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is 8 $m^3$ . The construction of the tank costs ₹ 70 per sq.m for the base and ₹ 45 per sq.m for sides.

(a) If $x$ and $y$ represent the length and breadth of the rectangular base, find the relation between the variables.

(b) Express the cost C of making the tank in terms of $x$.

(c) Find the value of $x$ so that the cost of construction is minimum.

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(c) Verify by second derivative test that cost is minimum at a critical point.

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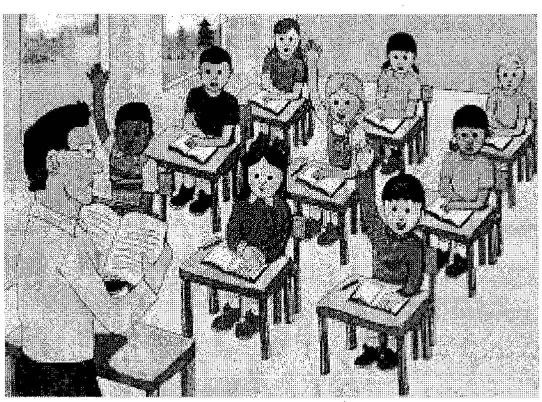

There are three categories of students in a class of 60 students :

A : Very hardworking students

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It is known that 10 students are in Category A, 30 in Category B and the rest in Category C. It is also found that the probability of students of Category A, unable to get good marks in the final exam, is 0.002, of Category B it is 0.02 and of Category C, this probability is 0.20.

(a) If a student selected at random was found to be the one who could not get good marks in the exam, find the probability that this student is of Category C.

(b) Find the probability that the selected student is unable to get good marks in the exam.

(c) A student selected at random was found to be the one who could not get good marks in the exam, then find the probability that this student is NOT of Category A.

(OR)

(c) If a student selected at random was found to be the one who could not get good marks in the exam, find the probability that this student is of Category B.

******END OF THE QUESTION PAPER******

| ROLL NUMBER | | | | |
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QP.Code:041/01/3

# INDIAN SCHOOL MUSCAT
## FIRST PRE BOARD EXAMINATION 2023
## MATHEMATICS (041)

CLASS : XII
DATE: 15.01.2023

TIME ALLOTED : 3 HRS.
MAXIMUM MARKS: 80

***GENERAL INSTRUCTIONS:***

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's** and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)**-type questions of 2 mark each.
4. **Section C** has 6 **Short Answer (SA)**-type questions of 3 mark each.
5. **Section D** has 4 **Long Answer (LA)**-type questions of 5 mark each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

**SECTION – A (Questions 1 to 20 carry 1 mark each)**

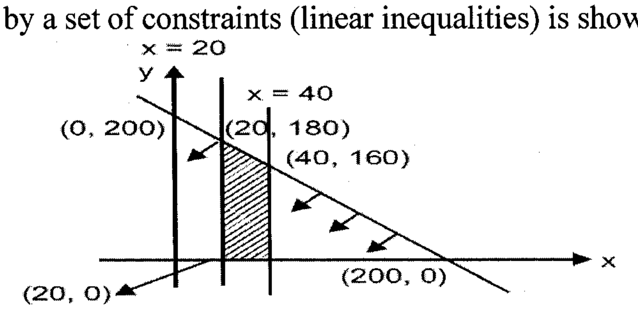
1. The point which does not lie in the half-plane $2x + 3y - 12 \leq 0$ is

(A)( 1, 2) (B) ( 2, 3) (C)( 2, 1) (D) ( -3, 2)

2. Let $A$ be a $3 \times 3$ matrix such that $C_{11}, C_{12}, C_{13}$ are the cofactors of $a_{11}, a_{12}, a_{13}$ respectively. What is the value of $a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$ ?

(A) 0 (B)1 (C) $-|A|$ (D) $|A|$

3. For an L.P.P. the objective function is Z = 400x + 300y, and the feasible region determined by a set of constraints (linear inequalities) is shown in the graph.




Find the coordinates at which the objective function is maximum.

(A)(20, 0) (B) (40, 0) (C) (40, 160) (D) (20, 180)

4. The greatest integer function defined by $f(x) = [x], 0 < x < 2$ is not differentiable at

(A) $x = 0$ (B) $x = 1$ (C) $x = 2$ (D) any point

5. If $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{b}$ is a vector such that $\vec{a}.\vec{b} = |\vec{b}|^2$ and $|\vec{a} - \vec{b}| = \sqrt{7}$, then $|\vec{b}| =$

(A)$\sqrt{7}$ (B) $\sqrt{3}$ (C) 7 (D)3

6. If $A$ and $B$ are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A/B) = \frac{1}{4}$, then $P(A' \cap B')$ equals

(A) $\frac{1}{12}$ (B) $\frac{3}{4}$ (C) $\frac{1}{4}$ (D) $\frac{3}{16}$

7. If $A$ is a matrix of order $p \times q$ and $B$ is a matrix such that $A'B$ and $B'A$ are both defined, then the order of matrix $B'$ is

(A) $p \times q$ (B) $q \times q$ (C) $p \times p$ (D) $q \times p$

8. If $A = [a_{ij}]_{m \times n}$ is a square matrix, then which of the following is true?

(A) $m < n$ (B) $m > n$ (C) $m = n$ (D) $m = 0$

9. $\int x\, sin(3 - x^2)dx$ equals

(A) $-\frac{1}{2}cos(3 - x^2) + C$ (B) $\frac{1}{2}cos(3 - x^2) + C$

(C) $\frac{1}{2}sin\,(3 - x^2) + C$ (D) $-\frac{1}{2}\,sin(3 - x^2) + C$

10. If $A = [a_{ij}]$be a skew-symmetric matrix of order $n$, then

(A) $a_{ij} = \frac{1}{a_{ji}}$ for all $i, j$ (B) $a_{ij} \neq 0$ for all $i, j$

(C) $a_{ij} = 0$, where $i = j$ (D) $a_{ij} \neq 0$, where $i = j$

11. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then the value of $x$ is

(A) $\pm 6$ (B) 3 (C) 6 (D) $\pm 3$

12. The integrating factor of the differential equation$\frac{dy}{dx} + y\, tanx - secx = 0$ is

(A) $\tan x$ (B) $sec^2x$ (C) $\sec x$ (D)$\frac{tan^2x}{2}$

13. The order and the degree of the differential equation $2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ are:

(A) 1, 1 (B) 2, 1 (C) 1, 2 (D) 3, 1

14. If $f(x) = x\,\tan^{-1} x$, then$f'(1) =$

(A) $1 + \frac{\pi}{4}$ (B) $\frac{1}{2} + \frac{\pi}{4}$ (C) $\frac{1}{2} - \frac{\pi}{4}$ (D) 2

15. $\int \frac{sin^2x - cos^2x}{sin^2x\, cos^2x}\, dx$ is equal to

(A) $tanx + cosec\, x + C$ (B) $tanx + cot\, x + C$

(C) $tanx - cot\, x + C$ (D) $tanx + sec\, x + C$

16. A point that lies on the line $\frac{x-1}{-2} = \frac{y+3}{4} = \frac{1-z}{7}$ is:

(A) (1, -3, 1) (B) (-2, 4, 7) (C) (-1, 3, 1) (D)(2, -4, -7)

17. A man is watching an aero plane which is at the coordinate point A (4, -1, 3), assuming the man is at O (0, 0, 0). At the same time, he saw a bird at coordinate point B (2, 0, 4). The unit vector along $\overrightarrow{AB}$ is

(A) $\frac{2}{6}\hat{\imath} + \frac{1}{6}\hat{\jmath} + \frac{1}{6}\hat{k}$ (B) $\frac{-2}{\sqrt{6}}\hat{\imath} + \frac{1}{6}\hat{\jmath} + \frac{1}{6}\hat{k}$

(C) $\frac{-2}{\sqrt{6}}\hat{\imath} + \frac{1}{\sqrt{6}}\hat{\jmath} + \frac{1}{\sqrt{6}}\hat{k}$ (D) $\frac{4}{\sqrt{6}}\hat{\imath} + \frac{2}{\sqrt{6}}\hat{\jmath} + \frac{3}{\sqrt{6}}\hat{k}$

18. The value of $(\hat{k} \times \hat{\jmath}).\hat{\imath} + (\hat{\jmath}.\hat{k}) + 3$

(A) 0 (B) 1 (C) 2 (D) 3

**ASSERTION-REASON BASED QUESTIONS**

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both **A** and **R** are true and **R** is the correct explanation of **A**.

(b) Both **A** and **R** are true but **R** is not the correct explanation of **A**.

(c) **A** is true but **R** is false.

(d) **A** is false but **R** is true.

19. **Assertion (A) :** $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \frac{2\pi}{3}$

**Reason (R) :** $\sin^{-1}(\sin\theta) = \theta$, if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

20. **Assertion (A) :** The angle between the lines whose direction cosines are

$$-\frac{\sqrt{3}}{4}, \frac{1}{4}, -\frac{\sqrt{3}}{2} \text{ and } -\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2} \text{ is } 120°.$$

**Reason (R) :** The angle between the lines whose direction cosines are $l_1, m_1, n_1$ and $l_2, m_2, n_2$ is given by $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$.

**SECTION – B (Questions 21 to 25 carry 2 marks each)**

21. If $\theta$ is the angle between two vectors $\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$, find $\sin\theta$.

(OR)

Find the value of $\lambda$ for which the points $A(-1,3,2)$, $B(-4,2,-2)$ and $C(5,5,\lambda)$ are collinear.

22. Find the value of : $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right] + \cos^{-1}\left[\cos\left(\frac{13\pi}{6}\right)\right]$

23. If $f(x) = \sqrt{1 + \cos^2(x^2)}$, find $f'\left(\frac{\sqrt{\pi}}{2}\right)$.

OR

If f(x) = $\tan^{-1}\left(\frac{\sqrt{x^2+1}-1}{x}\right)$, then find f '(x)

24. Find the intervals in which the function $f(x) = 2x^3 - 24x + 107$ is

(i) strictly increasing (ii) strictly decreasing

25. Find the projection of vector $(\vec{b} + \vec{c})$ on $\vec{a}$, where $\vec{a} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$ .

**SECTION – C (Questions 26 to 31 carry 3 marks each)**

26. Evaluate $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$ using properties of determinants.

27. Amit and Nisha appear for an interview for two vacancies in a company. The probability of Amit's selection is $\frac{1}{5}$ and that of Nisha's selection is $\frac{1}{6}$ .What is the probability that

(i) only one of them is selected? (ii) none of them are selected ?

(OR)

Find the mean number of defective items if two items are drawn one-by-one without replacement from an urn containing 6 items, which include 2 defective items?

28. Solve the following Linear Programming Problem graphically:

Maximize Z = $x + y$, subject to the constraints

$\frac{x}{25} + \frac{y}{40} \leq 1$; $2x + 5y \leq 100, x \geq 0, y \geq 0.$

29. Find the particular solution of the differential equation : $\frac{dy}{dx} + 2y \tan x = \sin x$, given that $y = 0$ when $x = \frac{\pi}{3}$

(OR)

Solve the following differential equation: $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + xdy = 0$

30. Evaluate: $\int_1^4 [|x-1| + |x-2| + |x-4|]\ dx$

31. Evaluate : $\int \frac{3x+5}{x^3-x^2-x+1} dx$ (OR) Evaluate : $\int x^2 \tan^{-1} x\ dx$

## SECTION – D (Questions 32 to 35 carry 5 marks each)

32. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and the straight line $3x+4y=12$.

33. Let $Z$ be the set of all integers and $R$ be the relation on $Z$ defined as $R=\{(a,b): a,b \in Z \text{ and } (a-b)\ is\ divisible\ by\ 5\}$. Prove that $R$ is an equivalence relation.

(OR)

Let $f$: **N** →**N** be defined by
$f(x)=\begin{cases} x+1, if\ x\ is\ odd \\ x-1, if\ x\ is\ even \end{cases}$ for all $x \in$ N. Show that $f$ is bijective.

34. $If\ A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, Find $A^{-1}$. Using $A^{-1}$, solve the system of linear equations $x-2y=10,\ 2x-y-z=8, -2y+z=7.$

35. Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Also find their point of intersection.

(OR)

The scalar product of the vector $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$ with a unit vector along the sum of vectors $\vec{b}=2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ and $\vec{c}=\lambda\hat{\imath}+2\hat{\jmath}+3\hat{k}$ is equal to 1. Find the value of $\lambda$.

## SECTION – E- CASE STUDY (Questions 36 to 38 carry 4 marks each)

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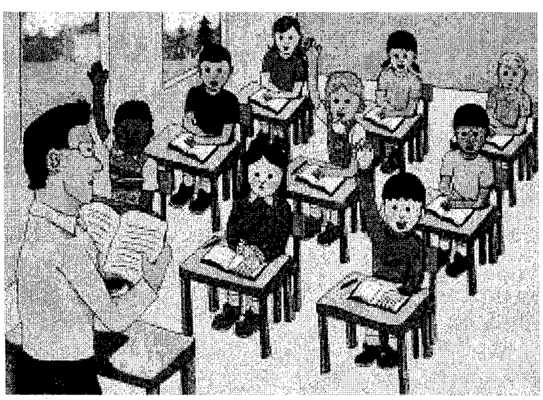

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(OR)

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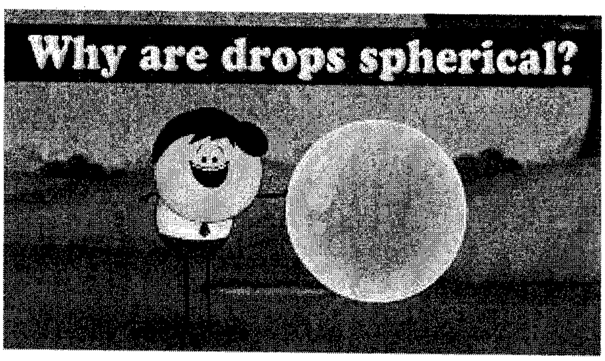
37.

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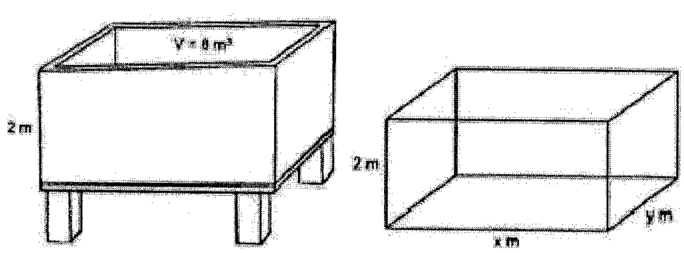

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(OR)

(c) Verify by second derivative test that cost is minimum at a critical point.

******END OF THE QUESTION PAPER******